

A simple method for electrical machine's mechanical parameter extraction

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ABSTRACT

Electrical motors are one of the most important key components of industry. While motors can be divided in so many groups, mathematical description of all of them is divided into two subsystems: Electrical and mechanical subsystem. Mechanical subsystem is usually composed of rotor inertia, external load and friction in bearing of rotor. Friction is usually modeled as viscous friction, i.e. linearly dependent on angular speed. Rotor inertia (J) and coefficient of viscous friction (B) are needed in order to model mechanical subsystem of motor. When motor is used in high performance close loop motion control systems, an accurate model of motor is required for system analysis and design. This paper suggests a novel method for measuring rotor's J and B for such applications. There is no restriction on the type of motor under test. Studied method, needs no sensor so no friction is added to motor. Only a digital camera is required. There is no need to open the motor case and remove motor's rotor in this method. Proposed method has been tested in laboratory and practical results shows effectiveness of suggested method.

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1. Introduction

Electrical energy comes from renewable and non-renewable sources. Some of the electrical energy's advantages over other forms of energy are as follows:

- Relatively cheap and available almost everywhere.
- Easiness of transport from one point to the other. Only a pair of wire is needed. Also, transfer of large amount of energy can be done with high efficiency.
- When converted to mechanical power, no smoke or other forms of air pollution is produced unlike internal combustion engines. So, electrical motors can work in closed environment such as homes and offices.
- Electrical energy can be converted to almost all other forms of energy easily.
- In addition to the aforementioned benefits, electrical energy has some drawbacks:
- Storing large amounts of electrical energy is not an easy task.

- When used improperly can lead to death.

Transformation of electrical power to mechanical power can be done with the aid of electrical motors. Fig. 1 shows different categories of electrical motors.

Although different names can be seen in Fig. 1, it must be emphasized that working principles of all these families are the same: A conductor of length l , carrying current I , experience a force F in presence of magnetic field B which can be calculated as: $F = I \cdot (l \times B)$.

Generally speaking, motor's generated torque must overcome two resistive counter torques: load's torque and friction. Equation of motion can be written as (Eq. 1):

$$\tau - \tau_{load} - B\omega = J \frac{d}{dt}(\omega) \quad (1)$$

where,

τ : Motor's generated torque,

τ_{load} : Load's torque

B : Coefficient of viscous friction,

ω : Load's angular speed,

J : Moment of inertia.

Developing a method for motor's mechanical parameter extraction (B and J in Eq. 1) is the aim of this paper.

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1.1. Literature review

There are several analytical and experimental methods to find moment of inertia in literature

(Boldea and Nasar, 1986; Vas, 2001; Cornea et al., 2007; Zverlov et al., 1999). Traditional methods are:

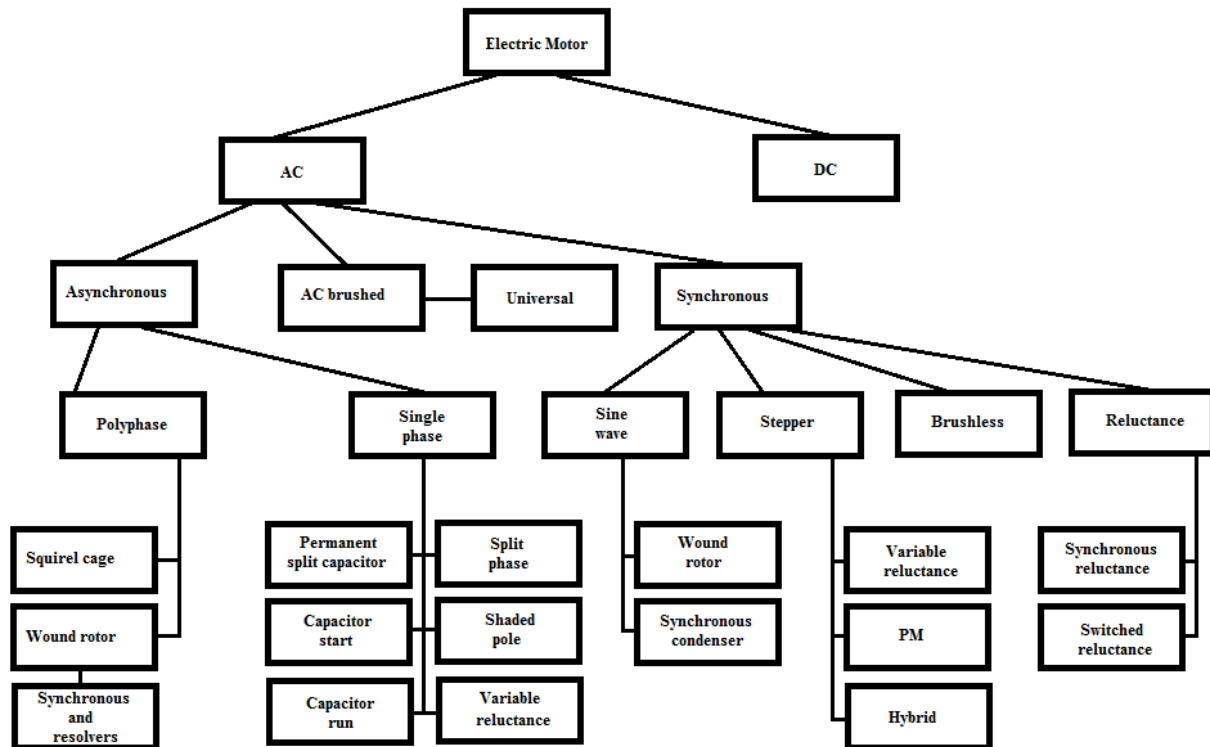


Fig. 1: Different categories of electrical motors

- Torsion oscillation test,
- Pendulum test,
- Retardation test.

The torsion oscillation test needs the removal of rotor. Rotor is hanging from a torsion wire and is rotated around rotor axes. Period of oscillation, determines the inertia. In practice, instead of aforementioned procedure, inertia of Motor under Test (MUT) is calculated using a comparison with a known rotor (Eq. 2) (Ilin, 2011):

$$J_{rotor} = J_{known} \times \left(\frac{T_{rotor}}{T_{known}} \right)^2 \quad (2)$$

Pendulum test and retardation test do not need rotor removal. In pendulum test, a bar of length l is connected to the rotor's shaft. Mass of m is connected to free end of bar. Bar can rotate around motor's shaft. Using period of oscillations (T), inertia can be calculated as (Eq. 3) (Ilin, 2011):

$$J_{rotor} = m \cdot l \cdot \left(\frac{T^2 \cdot g}{4 \cdot \pi^2} - l \right) \quad (3)$$

Retardation test is based on rotating the shaft up to a speed over the nominal speed. Machine is let free to slow down and speed is recorded with a data acquisition system.

After some algebra, inertia can be calculated using Eq. 4 (Ilin, 2011):

$$J_{rotor} = - \frac{P_{mn}}{\left(\frac{\pi}{30} \right)^2 \times n_n \times \frac{d}{dt}(n)_{n=n_n}} \quad (4)$$

M_{mn} is the torque corresponding to rated mechanical power losses, n_n is rated speed, $\frac{d}{dt}(n_n)$ represents the ramp of the line tangent AD in point A to the curve $n(t)$ and P_{mn} is the mechanical losses at rated speed. Fig. 2 shows the typical retardation test result for this method.

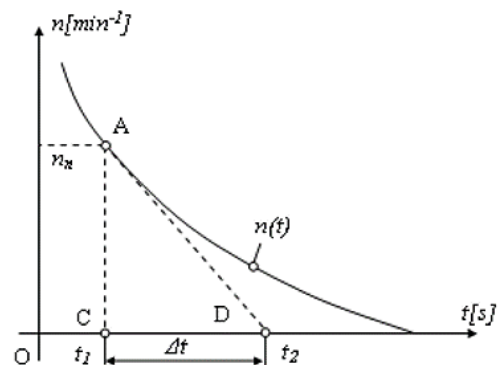


Fig. 2: Typical deceleration curve for retardation test

Also, more advanced techniques of inertia identification using observer (Awaya et al., 1992; Lee et al., 2004; Yang and Deng, 2005), extended Kalman filtering (Kwon et al., 2006; Boileau et al., 2008; Hong et al., 1996), model reference adaptive method (Andoh, 2008; Okamura et al., 1995), gradient descent method (Boileau et al., 2011) are available in

literature. These methods, generally needs a more complicated hardware.

2. Suggested method

In order to measure rotor moment of inertia (J) and coefficient of viscous friction (B) following two step procedures is suggested:

2.1. Measurement of $\frac{J}{B}$ Ratio

In this test, unloaded MUT is turned on and off is turned on and after reaching the nominal speed is turned off.

A pointer is attached to the motor's shaft. Shaft's deceleration is recorded with the aid of a digital camera. Obtained video is processed with video editing software so the deceleration interval's length (Interval between de energizing the motor and full stop of pointer attached to shaft) can be measured (ΔT_1).

When motor is de energized, equation of motion can be written as (Eqs. 5 and 6):

$$J \frac{d}{dt}(\omega) + B\omega = 0 \quad (5)$$

solution of Eq. 5 is:

$$\omega = \omega_0 \times e^{-\frac{B}{J}t} \quad (6)$$

Eq. 6 is approximately zero for $t \geq \frac{5 \times J}{B}$. So, $\Delta T_1 \approx \frac{5 \times J}{B}$ or $\frac{J}{B} \approx \frac{\Delta T_1}{5}$. So, ratio of $\frac{J}{B}$ can be approximated with the aid of this test.

2.2. Constant torque test

In this test, motor is disconnected from electrical source of energy and then a constant specific weight W is hanging from motor shaft with a short thread. Aim of this test is to apply a constant torque (τ_{thread}) to motor's shaft and study its motion under this condition.

Torque applied to motor's shaft can be calculated as Eq. 7:

$$\tau_{thread} = \frac{D}{2} \times W \quad (7)$$

Where D is motor's shaft diameter. If W is large enough rotor starts rotation. Rotor rotation is recorded with the aid of a digital camera. Using slow motion capability of video processing software, shaft's motion can be studied carefully. Pointer which is attached to motor's shaft and a dial make it possible to see shaft's position in each instant of time. Obtained video, is analyzed with video editing software in order to extract rotor's position as a function of time. Equation of motion in presence of a constant torque τ_{thread} is (Eqs. 8-10):

$$\tau_{thread} - B\omega = J \frac{d}{dt} \omega \quad (8)$$

$$\omega = \frac{\tau_{thread}}{B} (1 - e^{-\frac{B}{J}t}) \quad (9)$$

$$\omega = \frac{\tau_{thread}}{B} \left(1 - e^{-\frac{5}{\Delta T_1}t}\right) \quad (10)$$

If $\frac{B}{J}t \ll 1$ equivalently $t \ll \frac{B}{J}$ Eq. 10 can be approximated as (Eq. 11):

$$\omega \approx \frac{\tau_{thread}}{J} \times t \quad (11)$$

3. Extraction of parameters

After doing the above tests, mechanical parameters (B and J) can be extracted. As mentioned, $\frac{J}{B} \approx \frac{\Delta T_1}{5}$ or equivalently (Eq. 12):

$$J \approx \frac{\Delta T_1}{5} \times B \quad (12)$$

If B is known, J can be calculated using this equation. In order to calculate B, Eq. 10 and data obtained from constant torque test is used. So problem of finding B is converted to a curve fitting problem which can be solved using Least Square Method (LSM). After determining value of B using LSM, rotor's inertia can be calculated using Eq. 9.

4. Setup

In order to test aforementioned procedure, a digital camera, is used to record rotors motions. Dial is pasted on the motor's body. A water bottle connected with a thread to motor's shaft, is used as load (Fig. 3). Volume of the water in bottle is set such that rotor can start rotation. Motor can be de energized using a stop button. When motor's $\frac{J}{B}$ ratio is measured, camera records the moment which user press stop button so measurement of deceleration interval can be done with respect to the moment which energy is disconnected.



Fig. 3: Setup for constant torque test.

5. Practical results

Applying the aforementioned procedure to a 380 volt, 0.75 KW, 3-phase motor with $D=18.5$ mm lead to $\Delta T_1 = 6.74$ sec. Constant torque test is done with $W=281.19$ gr which lead to $\tau_{thread} = 0.0255$ N.m. Table 1 shows rotor's positions obtained for constant torque test. Also, Fig. 4, shows the plotted data.

Table 1: Rotor position vs. time for constant torque of 0.0255 N.M

No.	Time(sec.)	Shaft's angle (in Degrees)	Shaft's angle (in Radians)
1	0.000	0	0
2	0.135	0	0
3	0.383	5	0.087
4	0.570	9	0.157
5	0.645	14	0.244
6	0.780	21	0.367
7	0.858	24	0.419
8	0.925	29	0.506
9	1.053	38	0.663
10	1.110	43	0.751
11	1.248	55	0.960
12	1.323	60	1.047
13	1.390	65	1.135
14	1.458	75	1.309
15	1.518	80	1.396

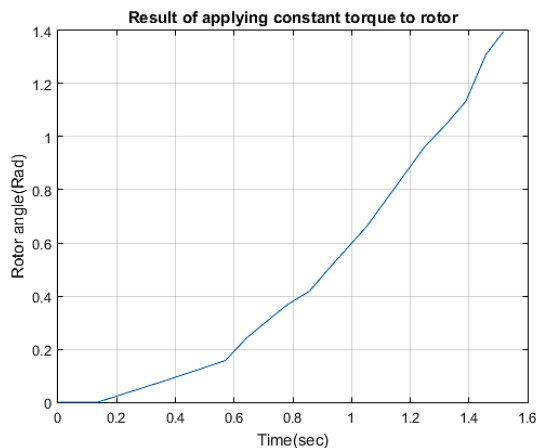


Fig. 4: Position of rotor in constant torque test

Fitting a second order polynomial to this data using Matlab® gives the following results (Eq. 13):

$$\theta = 0.6297 \times t^2 - 0.03138 \times t - 0.003571 \quad (13)$$

where rotor's angle (θ) is in radians and t is in seconds. Fig. 5 shows fitted curve and data points on the same graph. Result of curve fitting is shown in Fig. 6.

Using Eq. 13 angular speed can be found as (Eq. 14):

$$\omega = \frac{d\theta}{dt} = 1.2594 \times t - 0.03138 \quad (14)$$

Where $\omega = \frac{d\theta}{dt}$ is in $\frac{Rad}{s}$ and t is in seconds. We want to find the best coefficient of viscous friction (B) such that difference between Eqs. 10 and 14 is minimum. So, problem can be converted to an optimization problem:

Find B such that (Eqs. 10 and 14) is minimized.

Application of LSM to the above optimization problem leads to (Eq. 15):

$$B = 0.01025 \text{ N.m.s.} \quad (15)$$

and J can be calculated as (Eq. 16):

$$J = 0.0138 \text{ kg.m}^2. \quad (16)$$

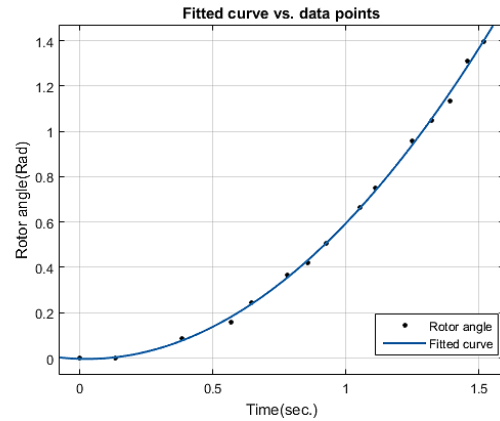


Fig. 5: Fitted curve vs. data points

Linear model Poly2:
 $f(x) = p1 \cdot x^2 + p2 \cdot x + p3$
 Coefficients (with 95% confidence bounds):
 $p1 = 0.6297 (0.5818, 0.6776)$
 $p2 = -0.03138 (-0.11, 0.04728)$
 $p3 = -0.003571 (-0.03239, 0.02525)$
 Goodness of fit:
 SSE: 0.003604
 R-square: 0.9989
 Adjusted R-square: 0.9987
 RMSE: 0.01733

Fig. 6: Results of curve fitting

Fig. 7, shows rotor's angular speed predicted by Eq. 14 vs. values predicted by best fit (Eq. 10 with $B=0.01025$ N.m.s) at time instances of Table 1. Fig. 8, shows statistical properties of best fit.

5.1. Verification of obtained results

In order to ensure correctness of the method, motor's case has been opened rotor's inertia has been calculated with the formulas of inertia for cylindrical sections. Fig. 9 shows motor's parts. Rotor's mass measured as 2.582 Kg. Rotor's inertia has been calculated as shown in Eq. 17.

$$B = 0.0015 \text{ Kg.m}^2 \quad (17)$$

This shows a tremendous error between what has been predicted by aforementioned method. In order to find the reason, look more carefully to model that has been assumed for rotor's motion which has been written here again for easiness of reference:

$$J \frac{d^2}{dt^2}(\theta) + B \frac{d}{dt}(\theta) = \tau_{thread} \quad \theta(0) = \dot{\theta}(0) = 0.$$

This equation suggests that there is a movement, i.e. change in rotor's angular position for all values of τ_{thread} . But this is not what seen in real world.

When a torque is applied to motor's shaft there is no movement up to a certain threshold torque ($\tau_{threshold}$).

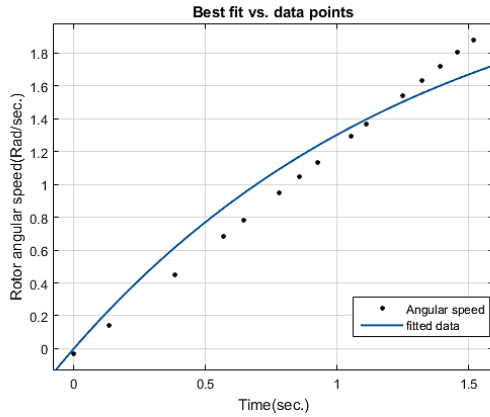


Fig. 7: Eq. 7 with $B=0.01025$ vs. Eq. 14

General model:
 $f(X) = .0255/B*(1-\exp(-.742*X))$
 Coefficients (with 95% confidence bounds):
 $B = 0.01025 (0.009658, 0.01084)$
 Goodness of fit:
 SSE: 0.2307
 R-square: 0.9539
 Adjusted R-square: 0.9539
 RMSE: 0.1284

Fig. 8: Statistical properties of best fit

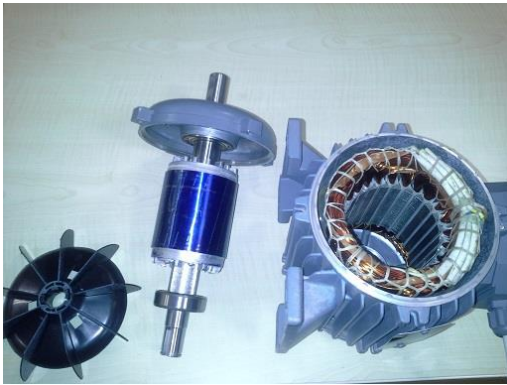


Fig. 9: Opening motor's case

When $w < 260 \text{ gr}$ (see Eq. 4) there is no rotation in the motor's shaft so when $w = 281.19 \text{ gr}$ is applied as in the above tests, applied torque to motor's shaft, which cause shaft's acceleration is not $0.28119 \times g \times \frac{D}{2} = 0.0255$ but $(0.28119 - 0.260) \times g \times \frac{D}{2} = 0.002 \text{ N.m}$ where $g = 9.81 \frac{m}{s^2}$ is gravitational field intensity and $D=18.5 \text{ mm}$ is motor's shaft diameter.

So, equation of motion can be corrected as (Eq. 18):

$$J \frac{d^2}{dt^2}(\theta) + B \frac{d}{dt}(\theta) = \tau_{thread} - \tau_{threshold} \quad \theta(0) = \dot{\theta}(0) = 0 \quad (18)$$

Here, aforementioned procedure is modified as follows.

5.2. Modification of method

First, Table 1 data is fitted on a Fourier model (Eq. 19):

$$\theta = 7.371 - 7.372 \times \cos(0.4252 \times t) - 0.1434 \times \sin(0.4252 \times t) \quad (19)$$

Data points of Table 1 and Eq. 19, are shown on the same graph in Fig. 10, also statistical properties of fit can be seen in Fig. 11.

Using Eq. 18, motion equation can be written in matrix form as (Eq. 20):

$$\left[\frac{d}{dt^2}(\theta) \quad \frac{d}{dt}(\theta) \right] \times \begin{bmatrix} J \\ B \end{bmatrix} = [\tau_{thread} - \tau_{threshold}] \quad (20)$$

Matrix $R = \begin{bmatrix} \frac{d}{dt^2}(\theta) & \frac{d}{dt}(\theta) \end{bmatrix}$ is repressor matrix and $P = \begin{bmatrix} J \\ B \end{bmatrix}$ parameter matrix. According to Table 1, 15, there are data points $t_1, t_2, t_3, \dots, t_{15}$. If R is calculated for $t_1, t_2, t_3, \dots, t_{15}$ with θ given by Eq. 20, a 15×2 matrix is obtained (Eq. 21):

$$R = \begin{bmatrix} 1.3328 & -0.0610 \\ 1.3321 & 0.1190 \\ 1.3194 & 0.4474 \\ 1.3001 & 0.6931 \\ 1.2900 & 0.7902 \\ 1.2686 & 0.9630 \\ 1.2545 & 1.0608 \\ 1.2410 & 1.1450 \\ 1.2128 & 1.3015 \\ 1.1989 & 1.3708 \\ 1.1628 & 1.5332 \\ 1.1414 & 1.6196 \\ 1.1212 & 1.6960 \\ 1.1000 & 1.7710 \\ 1.0805 & 1.8364 \end{bmatrix} \quad (21)$$

Also $C = [\tau_{thread} - \tau_{threshold}]$ is a constant 15×1 matrix.

$$C = \begin{bmatrix} 0.002 \\ 0.002 \\ 0.002 \\ 0.002 \\ 0.002 \\ 0.002 \\ 0.002 \\ 0.002 \\ 0.002 \\ 0.002 \\ 0.002 \\ 0.002 \\ 0.002 \\ 0.002 \\ 0.002 \end{bmatrix} \quad (22)$$

As seen from Eq. 20, there are only 2 unknowns where there are 15 equations. This system can't be solved using matrix inversion techniques because coefficient matrix is not a square matrix. Instead, Moore - Penrose pseudo inverse can be used (Eq. 23):

$$\tilde{R} = (R^t \times R)^{-1} \times R^t \quad (23)$$

where, \tilde{R} and R^t shows Moore – Penrose pseudo inverse of R and transpose of matrix R , respectively. So, matrix P can be calculated as (Eq. 24):

$$P = \begin{bmatrix} J \\ B \end{bmatrix} = \tilde{R} \times C \quad (24)$$

Following results are obtained after doing the calculations (Eq. 25):

$$P = \begin{bmatrix} J \\ B \end{bmatrix} = \begin{bmatrix} 0.0015 \\ 0.0002 \end{bmatrix} \quad (25)$$

Obtained value for J is the same as Eq. 17.

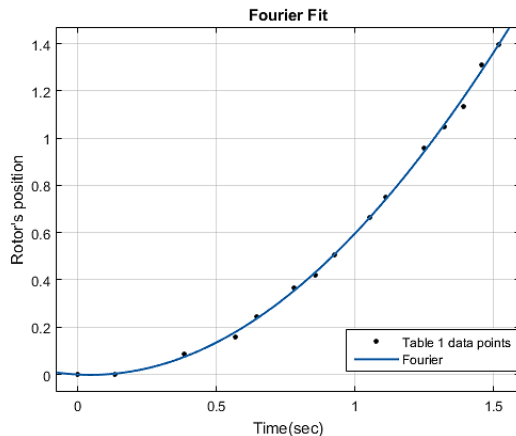


Fig. 10: Fitting Table 1 data to a sinusoidal model

General model Fourier1:
 $f(x) = a_0 + a_1 \cos(x \cdot w) + b_1 \sin(x \cdot w)$
 Coefficients (with 95% confidence bounds):
 $a_0 = 7.371 \text{ } (-25.3, 40.04)$
 $a_1 = -7.372 \text{ } (-40.05, 25.31)$
 $b_1 = -0.1434 \text{ } (-0.3345, 0.04772)$
 $w = 0.4252 \text{ } (-0.5714, 1.422)$
 Goodness of fit:
 SSE: 0.003535
 R-square: 0.9989
 Adjusted R-square: 0.9986
 RMSE: 0.01793

Fig. 11: Statistical properties of Fourier fit

6. Conclusion

Motor's electrical and mechanical parameters are important in analyzing and designing High performance motion control systems. In this paper a novel method for motor's mechanical parameter extraction has been studied. There is no need to opening the motor's case and removal of the rotor in this method. No sensor is attached to motor under test so no external friction is added to the system. Proposed method is quite easy and cheap.

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